

Determination of the Phreatic Line in Sandy Soil Containing Gypsum under Groundwater Inundation

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Abstract. The rise of groundwater levels above the ground surface represents a serious risk to infrastructure, as it can undermine the stability of foundations supporting buildings, bridges, roads, and other essential systems. This challenge is intensifying due to the growing occurrence of extreme weather events and land-use changes that accelerate groundwater depletion and alter natural recharge patterns. Consequently, understanding groundwater inundation processes, their effects on soil–structure interaction, and establishing reliable approaches for vulnerability assessment and damage mitigation have become critical research priorities. In this study, a laboratory-scale physical model was developed to simulate groundwater seepage conditions and replicate field-like scenarios. The experimental program focused on monitoring seepage behavior through sandy soil mixed with varying proportions of gypsum under elevated groundwater table conditions. The principal aim was to establish a mathematical expression describing the phreatic line within this homogeneous soil system. A dimensionless parabolic function was derived and validated through statistical analysis, demonstrating strong predictive capability for estimating the phreatic surface. The investigation considered two main influencing factors: soil type (sandy soil with gypsum content) and groundwater table height. The proposed equation showed excellent agreement with experimentally measured phreatic lines, confirming its reliability and applicability.

Keywords: Phreatic line, groundwater, inundation, gypsum, sandy soil, modeling.

1. INTRODUCTION

Groundwater inundation is emerging as a critical global challenge, posing severe risks to infrastructure systems. The rising frequency of incidents has been strongly linked to adverse weather extremes, shifts in land use, and continuous groundwater extraction [1,2]. This phenomenon occurs when the water table exceeds the ground surface, producing detrimental effects on buildings, roads, bridges, and other essential infrastructure. The consequences are multifaceted, encompassing structural damage, economic losses, environmental degradation, and public health concerns. To build a comprehensive understanding of this problem, the present study reviews a wide range of sources including peer-reviewed articles, technical reports, conference proceedings, and other relevant literature. This review aims to establish the state of the art and highlight promising avenues for future research. In addition, a physical model was developed with the following objectives:

- To derive an equation describing the shape of the phreatic line of seepage through homogeneous sandy soil with gypsum content under high groundwater levels.
- To test and statistically validate a proposed parabolic curve equation for predicting the phreatic line.

The urgency of addressing groundwater inundation has been emphasized by recurrent extreme climatic events and intensified groundwater extraction, both of which exacerbate the vulnerability of infrastructure frameworks [3-5]. In their study, the authors recalibrated depth–damage metrics for homes exposed to various flood scenarios. The analysis incorporated multiple influencing parameters, including access to flood alerts, the interaction of ice with rapid water flows, flood duration, residents' prior flood experience, living space

dimensions, family income, and real estate value. Regression methodologies revealed significant variations in flood damage patterns and provided insight into the complex interplay of these factors.

Similarly, Merz et al. [6] examined the estimation uncertainty of direct monetary flood damage to buildings. Their analysis of approximately 4,000 records from nine flood events in Germany (1978–1994) highlighted that damage data follow a log-normal distribution, with variations persisting even when stratified by building type and inundation depth. Although the foundation model related overall damage to water depth, it failed to fully capture the variability caused by additional contributing factors. The study underscored inherent uncertainties in flood damage assessment and reinforced the need for improved data collection and prediction models.

Kelman and Spence [7], in an overview of flood actions on buildings, emphasized the mechanisms by which floods impact structural systems. Their work quantified immediate damages while also stressing the importance of addressing secondary damages and resilience measures. In a separate investigation, Vekerdy and Meijerink [8] studied groundwater level fluctuations induced by flooding within the alluvial aquifer of Kisalföld, northwestern Hungary. Their results demonstrated differences in time lags between river stages and groundwater responses for open versus semi-confined aquifers. Testing nine flood events, they evaluated the predictive capability of phreatic and confined condition equations, concluding that river stage dynamics and sediment deposition in floodplains substantially influence groundwater responses.

Rotzoll and Fletcher [9] examined the effects of sea-level rise on groundwater flooding in coastal zones, stressing the need for proactive adaptation and informed decision-making. More recently, Gold et al. [10] highlighted the exposure of more than two million people to tidal roadway submersion, underlining the growing risks to stormwater infrastructure and the intensification of urban flooding hazards. Within this context, the present study focuses on the seepage behavior of high groundwater levels through homogeneous sandy soil with gypsum content. Building on the work of Vazirani and Chandola [11], who noted that the phreatic line could be approximated by a parabolic curve, a dimensionless equation was developed through a laboratory physical model. The resulting formulation provides a reliable tool for estimating the phreatic surface under inundation conditions.

2. EXPERIMENTAL WORK

To examine the impact of elevated groundwater levels on soil behavior and foundation stability, a laboratory-scale physical model was designed and constructed [12]. The setup was developed to replicate seepage through sandy soil with varying gypsum content under controlled boundary conditions, allowing systematic observation of the phreatic line development and pore water response, see Figure 1.



Figure 1. Preparation of testing physical model.

The experimental apparatus consisted of a rectangular galvanized steel soil box with internal dimensions of 1.5 m in length, 0.3 m in width, and 0.5 m in height. The rigid steel frame ensured structural stability and minimized wall deformation during soil compaction and water infiltration.

- Transparent observation face: One side of the box was replaced with a 10-mm-thick acrylic plastic sheet to allow continuous visual monitoring of seepage patterns, soil saturation zones, and the phreatic line evolution.

- Piezometer instrumentation: The opposite face was fitted with 10 piezometer tubes (spaced at 0.15 m intervals along the length of the box and at varying depths) to measure pore water pressure distribution and determine the hydraulic gradient across the soil profile.
- Outlet system: An outlet pipe was installed at the base of the downstream end of the box to discharge excess water and maintain steady hydraulic flow conditions.

The soil box was hydraulically connected to a rectangular water tank measuring 0.3×0.3×1.0 m. One vertical face of the tank was perforated with 105 circular openings, each 2.1 cm in diameter, arranged in a uniform grid pattern. This perforated face acted as a diffusion panel, ensuring even distribution of water into the soil profile and simulating natural seepage through a porous boundary. By controlling the water level in the supply tank, different groundwater table elevations were imposed on the soil model. The box was filled with homogeneous sandy soil mixed with predefined proportions of gypsum to simulate weakly cemented natural deposits. The filling procedure followed a layered compaction technique:

- Soil was placed in lifts of approximately 15 cm thickness.
- Each layer was compacted manually using a steel rammer to achieve field-representative density conditions.
- The relative density was checked after each lift to ensure uniformity throughout the soil mass.

This approach reduced heterogeneity and improved reproducibility across test runs. The expected phreatic line was assumed to follow a parabolic trajectory based on analytical solutions (e.g., Dupuit–Forchheimer assumption), which would later be validated against experimental measurements from the piezometer readings and visual observations. The measurements and observations are:

- The water level in the supply tank was gradually raised to simulate rising groundwater conditions.
- Piezometer readings were recorded at each increment to establish the pore pressure distribution.
- Visual inspection through the transparent face was used to trace the actual phreatic line.
- Discharge at the outlet was measured to verify steady-state flow conditions.
- Data were compared with the predicted parabolic equation to assess accuracy.

In order to simulate field conditions within a controllable laboratory environment, a geometric scale factor (L_r) of 1:10 was adopted. This scaling approach was selected to realistically represent the depth of infrastructure foundations and the corresponding rise of groundwater seepage through the soil profile. By applying this reduced scale, the laboratory model was able to capture the essential hydraulic and geotechnical behavior while remaining practical in terms of construction and measurement. Prior to conducting the main seepage experiments, a comprehensive series of preliminary soil characterization tests was performed on the two types of soil considered in this study. These tests ensured that the physical and chemical properties of the soils were well defined, enabling accurate interpretation of experimental results. The laboratory investigations were carried out in accordance with recognized international standards, as summarized below:

- Soil classification and index properties: Determined following ASTM D421 procedures [13].
- Shear strength and mechanical properties: Evaluated based on ASTM D3080 [13].
- Chemical properties of soils: Conducted in accordance with B.S. 1377, Part 3 [14].
- Standard Proctor compaction and soil moisture-density relationships: Performed as per AASHTO T99 [15].
- Permeability test conducted according to ASTM D2434-19 [13].

The outcomes of these preliminary tests are presented in Table 1, providing the baseline geotechnical and chemical characteristics of the soils employed in the model. These properties were subsequently used to calibrate the experimental setup and interpret the observed phreatic line and seepage patterns under elevated groundwater conditions.

Table 1. Physical and chemical properties of sandy soil used in tests.

Physical properties		Chemical properties	
Property	Value	Property	Value
Percent of clay and silt	28%	Cl ⁻¹	3.55%
Percent of sand	71%	SO ₃	5.64%
Percent of gravel	1%	TSS	30.679%
Maximum dry density	1.90 gm/cm ³	Organic matter	1.28%
Optimum moisture content	9%	CaCO ₃	10.1%
Direct shear test	Cohesion	Gypsum content	12.23%
	Internal angle of friction	pH	8.8
Permeability			
	6.15×10 ⁻⁶ cm/s		

3. EXPERIMENTAL INVESTIGATION OF PHREATIC LINE

In the first series of experiments, a sandy soil with natural gypsum content was employed in order to replicate field conditions commonly encountered in arid and semi-arid regions. The experimental box was filled with the soil to a height of 0.50 m, with each layer compacted carefully to achieve uniform density and minimize heterogeneity. After preparation of the soil bed, a sequence of test runs was conducted by supplying water from the connected reservoir tank at different controlled water levels. The rise of groundwater within the soil mass was monitored using the plastic piezometers installed on the transparent face of the model. These piezometers were spaced at 15 cm intervals, allowing accurate observation and recording of the hydraulic head distribution across the soil profile. The recorded data were used to capture the progression of the phreatic line and the seepage characteristics under varying external water levels. The experimental outcomes are presented graphically and numerically. Figure 2 illustrates the relationship between the imposed water height in the reservoir tank and the corresponding groundwater levels recorded at different distances within the soil mass. Figure 3 provides a visual documentation of the testing operation, highlighting the experimental setup during water infiltration. The observed piezometer readings under different applied water heads are tabulated in Tables 2 and 3, where the observed groundwater heights (O) are listed for each experimental run.

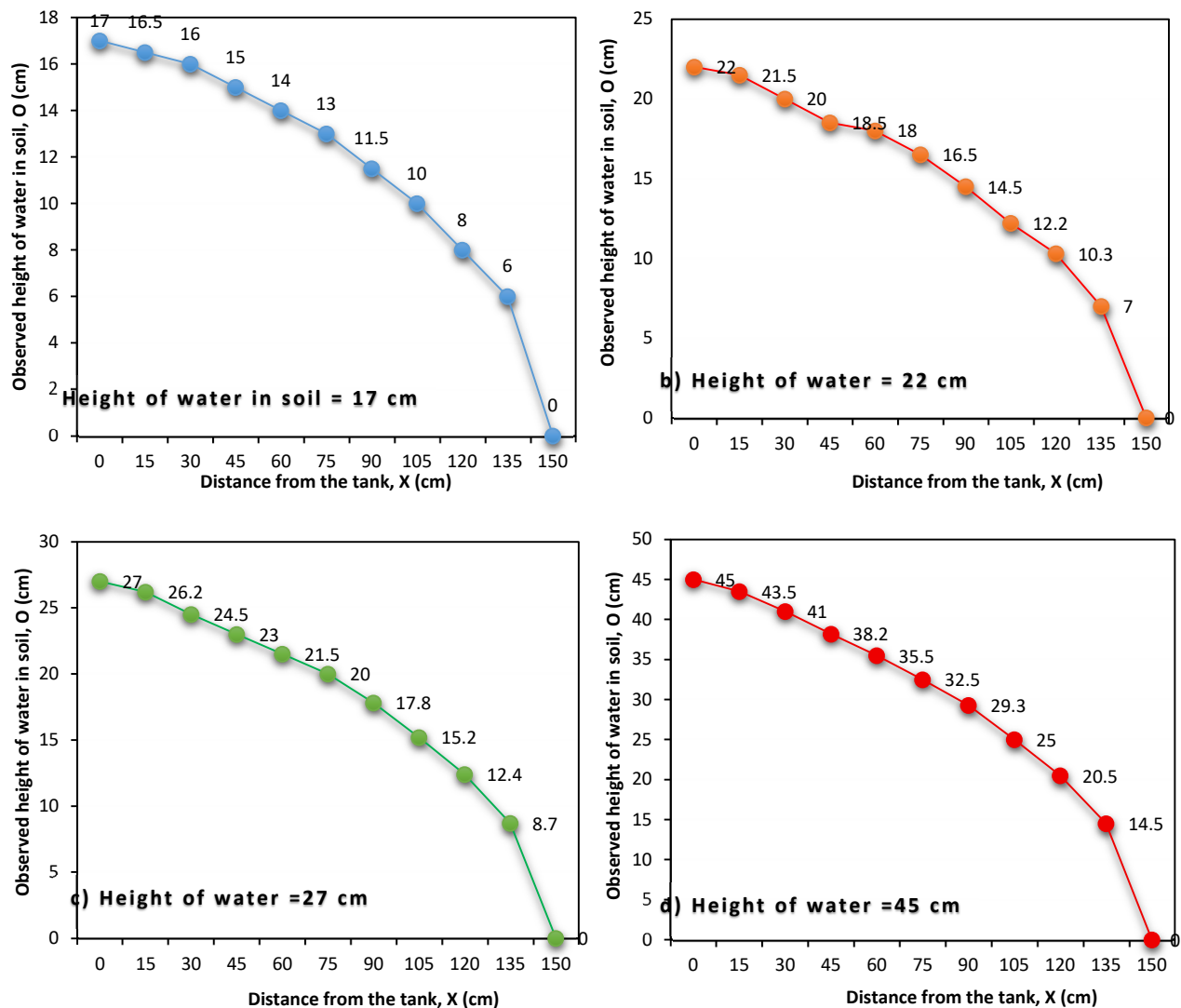


Figure 2. Observed heights of water (O) versus distances from the tank of water (X): a) $H = 17$ cm, b) $H = 22$ cm, c) $H = 27$ cm, and d) $H = 42$ cm.



Figure 3. Phreatic line measurements.

4. THEORETICAL INVESTIGATION OF PHREATIC LINE

Based on the observed groundwater levels recorded from the piezometers installed in the sandy soil model, the data indicated that the phreatic line could be reasonably approximated by a parabolic curve. This trend is consistent with the theoretical shape of the phreatic line in homogeneous soils, such as those observed in earth dams [11]. However, slight deviations were noted, which may be attributed to variations in soil properties and boundary conditions specific to the experimental setup. From these findings, a dimensionless governing equation was derived to describe the parabolic shape of the seepage line through the homogeneous sandy soil with gypsum content. This generalized relationship provides a practical predictive tool for estimating the position of the phreatic line under varying groundwater levels:

$$\frac{Y^2}{H^2} = 1 - \frac{X}{L} \quad (1)$$

Where

Y = The calculated height of water through soil at any distance (X) from tank.

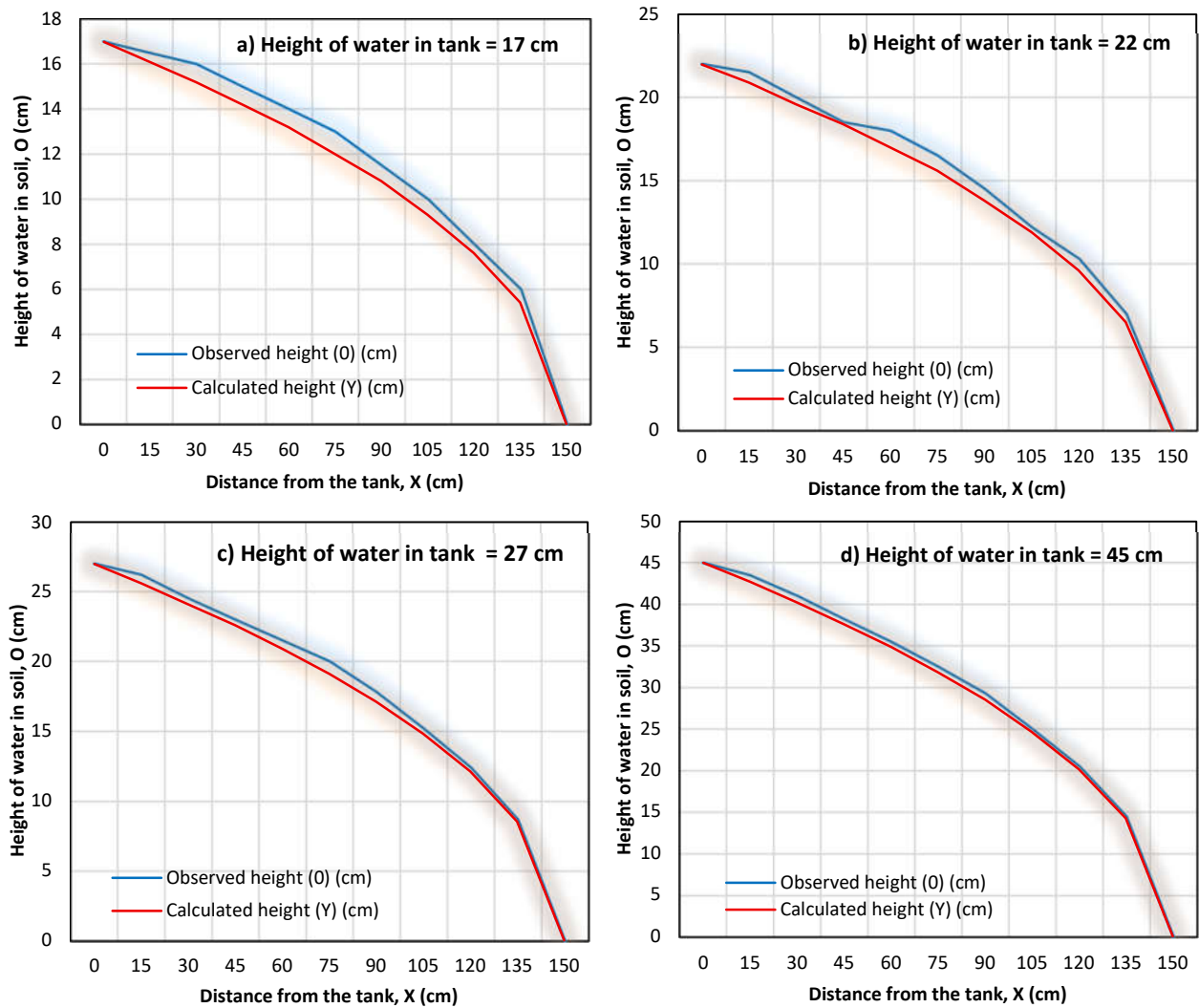
H = Height of water in the water tank.

X = Distance from the water tank.

L = Length of model (1.5 m)

Figure 4 illustrates the comparison between the observed water heights in the sandy soil model and the corresponding calculated values obtained using the estimated seepage Equation (1). The calculated results derived from Equation (1) are also provided in Tables 2 and 3, facilitating a direct evaluation of the agreement between experimental observations and theoretical predictions. Based on this comparison, the following key observations can be made:

- The differences between the observed seepage heights and the predicted values from Equation (1) were generally very small, confirming a strong correlation and high degree of agreement between experimental data and theoretical modeling. This validates the reliability of the proposed dimensionless parabolic relationship for predicting the phreatic line in homogeneous sandy soil with gypsum content.
- In all test cases, the observed readings consistently exceeded the corresponding calculated values. This deviation is primarily attributed to the effects of capillary rise, which are not fully represented in the theoretical seepage model. Capillary action tends to elevate the actual water level above the predicted phreatic surface, particularly in fine-grained sandy soils with gypsum content.
- Additional factors that may have contributed to these discrepancies include minor variations in soil density and compaction during model preparation, heterogeneity of particle size distribution within the sandy soil layers, and the sensitivity and resolution of piezometer measurements. Although these factors are secondary, they may have introduced slight deviations between the observed and predicted values.
- Despite these minor differences, the overall consistency between experimental and theoretical results strongly supports the applicability of Equation (1) for modeling seepage behavior in controlled soil systems.



Figures 4. Comparison of observed and calculated heights of water in soil using seepage equation: a) $H = 17$ cm, b) $H = 22$ cm, c) $H = 27$ cm, and d) $H = 42$ cm.

5. STATISTICAL ANALYSIS OF RESULTS

5.1 R-Squared Formula

R-squared (R^2) is a statistical metric that quantifies the proportion of variance in a dependent variable that can be explained by one or more independent variables in a regression model [16]. While correlation measures the strength and direction of a relationship between two variables, R^2 specifically indicates how much of the variability in the dependent variable is accounted for by the model. For example, an R^2 value of 0.5 suggests that approximately 50% of the observed variation in the dependent variable can be explained by the inputs included in the model.

$$R^2 = 1 - \frac{\text{Unexplained variation (SS}_{\text{res}})}{\text{Total variation (SS}_{\text{total}})} \quad (2)$$

$$SS_{\text{res}} = (O - Y)^2$$

$$SS_{\text{total}} = (O - \bar{O})^2$$

$$\bar{O} = \sum_{i=1}^n \frac{O}{n}$$

Where

O = Observed height of water in soil.

Y = Calculated height of water in soil.

$\bar{O}$ = mean value of observed height of water in soil.
 n = no. of points

The R-squared (R^2) values calculated for the height of water in sandy soil with gypsum content, corresponding to different water column heights, are presented in Tables 2 and 3. These results indicate that the observed data closely matched the predicted values, demonstrating a strong agreement between the experimental measurements and the estimated relationship.

Table 2. R – squared calculation for 17 and 22 cm height of water in sandy soil.

Point	Height of water 17 cm				Height of water 22 cm			
	O (cm)	Y (cm)	$SS_{res.} = (O - Y)^2$	$SS_{total} = (O - \bar{O})^2$	O (cm)	Y (cm)	$SS_{res.} = (O - Y)^2$	$SS_{total} = (O - \bar{O})^2$
1	17.0	17.0	0.00	29.76	22.0	22.0	0.00	54.9
2	16.5	16.1	0.16	24.55	21.5	20.9	0.36	47.7
3	16.0	15.2	0.64	19.85	20.0	19.6	0.16	29.3
4	15.0	14.2	0.64	11.94	18.5	18.4	0.01	15.3
5	14.0	13.2	0.64	6.03	18.0	17.0	1.00	11.6
6	13.0	12.0	1.00	2.12	16.5	15.6	0.81	3.6
7	11.5	10.8	0.49	0.002	14.5	13.8	0.49	0.008
8	10.0	9.3	0.49	2.39	12.2	11.9	0.09	5.7
9	8.0	7.6	0.16	12.57	10.3	9.6	0.49	18.4
10	6.0	5.4	0.36	30.75	7.0	6.5	0.25	57.6
11	0.0	0.0	0.00	133.29	0.0	0.0	0.00	212.9
11	127		4.480	273.25	160.5		3.66	457.0
$R^2 = 0.98$					$R^2 = 0.99$			

Table 3. R – squared calculation for 27 and 45 cm height of water in sandy soil.

Point	Height of water 27 cm				Height of water 45 cm			
	O (cm)	Y (cm)	$SS_{res.} = (O - Y)^2$	$SS_{total} = (O - \bar{O})^2$	O (cm)	Y (cm)	$SS_{res.} = (O - Y)^2$	$SS_{total} = (O - \bar{O})^2$
1	27.0	27.0	0.00	83.72	45	45	0	238.7
2	26.2	25.6	0.36	69.72	43.5	42.7	0.64	194.6
3	24.5	24.1	0.16	44.22	41.0	40.2	0.64	131.1
4	23.0	22.6	0.16	26.52	38.2	37.6	0.36	74.8
5	21.5	20.9	0.36	13.32	35.25	34.85	0.42	35.8
6	20.0	19.1	0.81	4.62	32.5	31.8	0.49	8.7
7	17.8	17.1	0.49	0.003	29.3	28.5	0.64	0.06
8	15.2	14.8	0.16	7.02	25.0	24.6	0.16	20.7
9	12.4	12.1	0.09	29.70	20.5	20.1	0.16	81.9
10	8.7	8.5	0.04	83.72	14.5	14.2	0.09	226.5
11	0	0	0	318.62	0	0	0	873.2
11	196.3		2.63	618.18	325		3.6	1885.6
$R^2 = 0.996$					$R^2 = 0.998$			

5.2 Root Mean Square Error (RMSE)

The Root Mean Square Error (RMSE) is a key performance metric for regression models, quantifying the average deviation between predicted values and actual observations. It provides an estimate of the model's predictive accuracy, with lower RMSE values indicating better model performance [17]. A perfect model, which would predict the exact observed values in all cases, would have an RMSE of zero [18]. One advantage of RMSE is that it is expressed in the same units as the predicted variable, making interpretation straightforward. RMSE is calculated using the following formula:

$$RMSE = \sqrt{\frac{1}{n} \sum_{i=1}^n (O - Y)^2} \quad (2)$$

Where

n = Number of non – missing data points.

O = Actual observation.

Y = Calculated points.

The RMSE values, calculated using Eq. 2 for sandy soil with gypsum content under varying water column heights, are presented in Table 4.

Table 4. RMSE for height of water 17 cm in sandy soil.

Point	Height of water 17 cm			Height of water 22 cm			Height of water 27 cm			Height of water 45 cm		
	O (cm)	Y (cm)	(O - Y) ²	O (cm)	Y (cm)	(O - Y) ²	O (cm)	Y (cm)	(O - Y) ²	O (cm)	Y (cm)	(O - Y) ²
1	17.0	17.0	0.00	22.0	22.0	0.00	27	27	0	45	45	0
2	16.5	16.1	0.16	21.5	20.9	0.36	26.2	25.6	0.36	43.5	42.7	0.64
3	16.0	15.2	0.64	20.0	19.6	0.16	24.5	24.1	0.16	41	40.2	0.64
4	15.0	14.2	0.64	18.5	18.4	0.01	23	22.6	0.16	38.2	37.6	0.36
5	14.0	13.2	0.64	18.0	17.0	1.0	21.5	20.9	0.36	35.5	34.85	0.42
6	13.0	12.0	1.00	16.5	15.6	0.81	20	19.1	0.81	32.5	31.8	0.49
7	11.5	10.8	0.49	14.5	13.8	0.49	17.8	17.1	0.49	29.3	28.5	0.64
8	10.0	9.3	0.49	12.2	11.9	0.09	15.2	14.8	0.16	25	24.6	0.16
9	8.0	7.6	0.16	12.3	9.6	0.49	12.4	12.1	0.09	20.5	20.1	0.16
10	6.0	5.4	0.36	7.0	6.5	0.25	8.7	8.5	0.04	14.5	14.2	0.09
11	0.0	0.0	0.00	0.0	0.0	0.00	0	0	0	0	0	0
11			4.58			3.66			2.63			3.6
RMSE = 0.65 (0-1)			RMSE = 0.58 (0-1)			RMSE = 0.49 (0-1)			RMSE = 0.57 (0-1)			

- RMSE is not scale-invariant; therefore, comparisons of models using this metric can be influenced by the scale of the data. For this reason, RMSE is often applied to standardized data.
- Since RMSE squares the errors before averaging, it can be disproportionately affected by a few large deviations. If minimizing the influence of extreme errors is desired, using absolute residuals or the median of errors can provide a more representative assessment of model performance.

Despite these considerations, the RMSE values for the water heights in sandy soil ranged from 0.49 to 0.72, which falls within the theoretical range of 0–1. This indicates that the estimated relationship expressed by Eq. 1 aligns well with the observed values obtained from the laboratory model tests.

6. CONCLUSIONS

Based on the experimental investigations and analyses conducted in this study, the following key conclusions can be drawn:

- The laboratory-scale investigation enabled the derivation of a dimensionless equation describing the phreatic line and seepage behavior of high groundwater levels through homogeneous sandy soil containing gypsum. This equation accurately represents the relationship between water height and distance in the soil, as validated against observed measurements from the model. The results demonstrate that the parabolic approximation can be effectively applied to similar homogeneous soils under controlled conditions.
- Observed deviations between the measured and predicted water heights, which were slightly higher in the experimental readings, are attributed primarily to capillary action and other minor soil-structure interactions. Despite these differences, the high R^2 values (close to 1) and low RMSE values (0.49–0.72) indicate that the proposed equation reliably predicts the phreatic surface in the laboratory model.
- It is recommended to conduct field-scale studies to validate the laboratory findings under natural environmental conditions. Such validation would help determine the influence of heterogeneities in soil properties, layered soil profiles, and boundary conditions that are not fully captured in the laboratory model.
- The study also suggests integrating the experimental results with theoretical models of seepage through porous media. This would allow comparison between predictive simulations and measured behavior, improving confidence in the model's applicability for real-world infrastructure, including foundations, roads, bridges, and embankments susceptible to high groundwater levels.
- Finally, the developed methodology provides a framework for assessing groundwater inundation risks and designing mitigation strategies for infrastructure exposed to fluctuating water tables. By combining laboratory modeling, field validation, and theoretical modeling, future studies can enhance predictive capabilities and support sustainable infrastructure management under conditions of groundwater rise and climate-related hydrological variability.

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